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Discontinuous differential equations: comparison of solution definitions and localization of hidden Chua attractors

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Abstract: This paper studies a class of systems with discontinuous right-hand side, which is commonly used in various applications. The notion of discontinuous system is closely linked to the notion of differential inclusion, which was first considered by Marchaud and Zaremba. In this paper three different notions of solutions of differential equations will be considered: Filippov, Aizerman-Pyatnitskiy and Gelig-Leonov-Yakubovich solutions. For the class of systems considered in the paper it is discussed when these definitions coincide and when they differ. The application of definitions is also demonstrated by numerical modelling of hidden attractor in Chua's circuit.

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1. DEFINITIONS OF SOLUTION OF DISCONTINUOUS SYSTEM

Consider the following system with discontinuous right-hand side:

$$\frac{dx}{dt} = Ax + b\psi(\sigma), \quad \sigma = c^* x, \tag{1}$$

where $x \in \mathbb{R}^n$, A is constant square matrix of order n, b, c are vectors of order n, $\sigma \in \mathbb{R}$ is the input of $\psi(\sigma)$, which is a piecewise continuous function in $\mathbb{R}$. If $\psi(\sigma)$ is continuous in a certain region, Peano theorem guarantees the existence of solution $x(t)$ of system (1) nearby t_0, which satisfies $x(t_0) = x_0$, where x_0 is a point of continuity region. However, the question on how to define a solution when ψ is discontinuous arises. Let Σ be the set of discontinuity points of the function $\psi(\sigma)$. In the current article we will consider discontinuities of the first order, i.e. the function $\psi(\sigma)$ is continuous for values σ, which are close to σ_0, and there exist finite limits

$$\lim_{\sigma \to \sigma_0 + 0} \psi(\sigma) = \psi(\sigma_0 + 0) \tag{2}$$

and

$$\lim_{\sigma \to \sigma_0 - 0} \psi(\sigma) = \psi(\sigma_0 - 0). \tag{3}$$

Nowadays there are many definitions of solutions of discontinuous systems (systems with discontinuous right-hand side) and numerous works are devoted to this subject (Marchaud, 1934; Zaremba, 1936; Carathéodory, 1918; Filippov, 1960; Aizerman and Pyatnitskiy, 1974; Gelig et al., 1978; Krasovskiy and Subbotin, 1974; Utkin, 1992; Cortes, 2008; Hui et al., 2009; Dieci and Lopez, 2012; Orlov, 2008; Boiko, 2008; Luo, 2012; Akhmet, 2010; Bernardo et al., 2008; Fečkan, 2011). All of these definitions are common in the following: the value of discontinuous function at each point is replaced by a set. In our case, the function $\psi(\sigma)$ is replaced by a multiple-valued function $\phi(\sigma)$. Note that in continuity points of function $\psi(\sigma)$ multiple-valued function $\phi(\sigma)$ consists of one point and coincides with $\psi(\sigma)$. In the points of discontinuity $\phi(\sigma)$ is a set, which is defined in a certain way.

Definition 1. Solution of equation (1) is an absolutely continuous vector-function $x(t)$, defined on a segment or interval I for which

$$\frac{dx}{dt} \in Ax + b\phi(\sigma), \quad \sigma = c^* x, \tag{4}$$

almost everywhere on I.

Expression (4) is called a *differential inclusion.*

To the best of our knowledge Definition 1 was suggested for the first time in (Gelig et al., 1978) and then repeated in (Filippov, 1985).

How does one define a range $\phi(\sigma_0)$ of the function $\psi(\sigma)$ in point of discontinuity $\sigma_0 \in \Sigma$ in order for the obtained solution to satisfy the similar basic properties of differential equations and also to have a physical meaning?

Let us describe three different definitions, which one will consider further.

The first definition was introduced by Filippov A.F. in 1960 and is also called "convex definition" (Filippov, 1960, 1985).

Definition 2. (Filippov). Suppose that for each point σ the function $\phi(\sigma)$ is a minimal closed convex set, which contains values of the vector function $\psi(\sigma')$ for $\sigma' \notin \Sigma$, $\sigma' \to \sigma$.

Solution $x(t) : I \to \mathbb{R}^n$ of system (4) is called *Filippov solution.*

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